

de Sitter vacua from uplifting D-terms in effective supergravities from realistic strings

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ABSTRACT: We study the possibility of using the D-term associated to an anomalous $U(1)$ for the uplifting of AdS vacua (to dS or Minkowski vacua) in effective supergravities arising from string theories, particularly in the type IIB context put forward by Kachru, Kallosh, Linde and Trivedi (KKLT). We find a gauge invariant formulation of such a scenario (avoiding previous inconsistencies), where the anomalous D-term cannot be cancelled, thus triggering the uplifting of the vacua. Then, we examine the general conditions for this to happen. Finally, we illustrate the results by presenting different successful examples in the type IIB context.

KEYWORDS: Flux compactifications, dS vacua in string theory, Supergravity Models.

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1. Introduction

Our increasingly precise knowledge of the evolution of the universe, condensed in the Λ CDM model, has confirmed inflation as a cornerstone of modern cosmology. This was an epoch of accelerated, de Sitter (dS) expansion in the distant past. There is also strong observational evidence that the universe is entering a second phase of accelerated expansion today, suggesting a positive vacuum energy at present [1].

It would be desirable that this progress in the area of cosmology were matched by analogous developments on the Particle Physics side. The outstanding problem there is the development of a consistent quantum theory of the fundamental interactions that would explain cosmological observations. It is widely believed that string theory fulfils all necessary requirements to become such fundamental theory, but there are a number of technical issues that are hampering progress. In particular, it is very difficult to find stable dS vacua, which is directly related to the problem of moduli stabilisation.

Recently, we have witnessed outstanding progress in this area, perhaps best represented by the results of Kachru et al. [2], known from now on as KKLT. They devised a method for constructing dS vacua through a combination of D-branes, fluxes and non-perturbative effects, which we will explain in detail in the next section, as a preamble to our work. The idea was, in the context of type IIB theory, to stabilise most of the moduli by fluxes at a high scale, and to write down an effective low-energy theory for the remaining, so far flat, T -modulus. This would be stabilised by adding non-perturbative effects to the superpotential. The resulting vacuum happens to be purely supersymmetric (SUSY) and

anti deSitter (AdS). The idea, then, is to add the effect of anti D3-branes to the model, which would act as an extra piece in the potential, breaking SUSY explicitly and uplifting the vacuum to dS.

There are a number of issues raised about this mechanism and its successful implementation within a realistic string model. In particular, the explicit supersymmetry breaking introduced by the anti D3-brane makes it difficult to compute corrections reliably, so it would be desirable to obtain dS vacua without explicit SUSY breaking. A suggestion by Burgess et al. [3], BKQ from now on, consisted of dropping the anti-D3 branes and considering D-terms coming from an anomalous U(1) instead. In favourable conditions the D-terms could act as an uplifting potential. This approach was subsequently criticised by Choi et al. [4] and de Alwis [5]. Quoting old results in Supergravity, a model with vanishing F-terms must have also vanishing D-terms. Therefore it is imperative that, in order for the D-term to act as an uplifting potential, the F-terms have to necessarily break SUSY (which was not the case for BKQ).

It is this whole issue that we want to address in the present letter. We will reconcile the results of BKQ, on the one hand, and Choi and de Alwis on the other, by exploiting the need for the superpotential to be gauge invariant once an anomalous U(1) is considered in the model.¹ We will show that a gauge invariant version of BKQ can lead to stable dS vacua with realistic values of the parameters in specific examples of type IIB compactifications.

The paper is organised as follows: in section 2, we give a comprehensive review of the structure of the KKLТ mechanism and the subsequent proposal of BKQ. In sections 3 and 4 we look at the consequences of implementing gauge invariance in a consistent way. In section 5 we give a summary of the anomaly cancellation conditions one has to take into account to work out consistent examples. In section 6 we present viable examples in the context of type IIB theory and, in section 7, we conclude.

2. Background

As already mentioned in the introduction, KKLТ found an explicit way to construct 4D de Sitter solutions of string theory. Their proposal was to use background fluxes for both NS and RR forms to fix the complex-structure moduli of a Calabi-Yau compactification in the context of type IIB theory. They considered just one overall Kähler modulus, T , not fixed by the fluxes, which they stabilise through non-perturbative effects. In the language of N=1 Supergravity (SUGRA), this is equivalent to considering a model with Kähler potential

$$K = -3 \log(T + \bar{T}) \quad , \quad (2.1)$$

and superpotential

$$W = W_0 + Ae^{-aT} \quad . \quad (2.2)$$

W_0 is an effective parameter, coming from having integrated out all complex-structure moduli through the use of fluxes [6], and A, a are constants. The non-perturbative superpotential is either generated by Euclidean D3-branes or by gaugino condensation [7–9] in

¹Actually, BKQ mentioned this point in their article but, for simplicity, they did not attempt to incorporate gauge invariance in the analysis.

a non-abelian sector of N wrapped D7-branes. Notice here that the gauge kinetic functions in the effective SUGRA theory are given (up to $\mathcal{O}(1)$ factors, see sect. 5 below) by

$$f_a = \frac{T}{2\pi}. \tag{2.3}$$

The corresponding scalar potential [given just by the F-part, V_F , see eqs (2.8) below] has a SUSY-preserving, AdS minimum for T , which thus gets stabilised, but at a negative energy. In order to promote these AdS vacua to dS vacua, the authors add anti D3-branes to the construction, which they quantify in terms of a new piece in the potential, namely

$$V = V_F + \frac{k}{T_R^2}, \tag{2.4}$$

where k is a constant and $T_R = \text{Re } T$. If k acquires a suitable value, then this term can uplift the AdS vacuum obtained just with V_F . Notice that the k/T_R^2 piece breaks supersymmetry explicitly which, besides aesthetic reasons, complicates the analysis and reduces our control over the effective theory.

In ref. [3] BKQ proposed an attractive variation of KKLT to avoid such shortcomings. Namely, instead of anti D3-branes, they considered the possibility of turning on fluxes for the gauge fields living on D7-branes. These fluxes would, in turn, generate a Fayet-Iliopoulos (FI) term in the 4D effective action of the form

$$V_D = \frac{1}{2g_{\text{YM}}^2} D^2 = \frac{\pi}{T_R} \left(\frac{E}{T_R} + \sum_I q_i |\varphi_i|^2 \right)^2, \tag{2.5}$$

where $T_R = 2\pi/g_{\text{YM}}^2$, E is some constant arising from non-trivial fluxes for the gauge fields living on the D7-branes, φ_i are the scalar components of the chiral superfields, except T , and q_i are the corresponding charges under the anomalous $U(1)_X$ group. The previous equation assumes a minimal Kähler potential for φ_i , which is clearly a simplification, but it does not affect crucially the results.

The next assumption of BKQ is that (at the minimum) all $\langle \varphi_i \rangle = 0$. They argue that this can arise in two different ways: if the q_i charges are such that the above D-term cannot be cancelled, it is reasonable to assume that the φ_i fields will settle at the origin to minimise V . As the authors admit, this “non-cancellability” is a crucial assumption which, so far, has not proved to be realised in practice. Alternatively, if the superpotential does not depend on the φ_i fields [as happens in eq. (2.2)], then the corresponding F-term contributions to the potential, $|D_i W|^2 \sim |\varphi_i W|^2$ may be efficient to render $\varphi_i = 0$ at the minimum. In either case, they can replace

$$V_D \sim \frac{\pi E^2}{T_R^3}, \tag{2.6}$$

which has a form similar to the non-SUSY potential suggested by KKLT in (2.4) and, indeed, can work as an uplifting term: by adding the piece (2.6) with an appropriate value of E to the V_F potential, the initial KKLT AdS minimum becomes a dS vacuum, as desired.

The results of BKQ have been criticised by Choi et al. [4] (and also in ref. [5]). Their argument is based on the simple relation between the value of the D-terms and the F-terms in a (D=4, N=1) SUGRA theory, namely

$$D^a = \frac{i}{\text{Re}(f_a)} \frac{1}{W} \eta_a^I D_I W \quad , \quad (2.7)$$

where Φ^I are the chiral superfields, $\eta_a^I(\Phi)$ denote their gauge transformation, $\delta\Phi^I = \epsilon_a \eta_a^I(\Phi)$ (ϵ_a are the infinitesimal parameters of the transformation), $D^a = i\eta_a^I K_I / \text{Re}(f_a)$, and D_I is the Kähler derivative: $D_I W = \partial_I W + (\partial_I K)W$. ($X_I \equiv \partial_I X = \partial X / \partial \Phi_I$, as usual).

Eq. (2.7) is valid at any point in field space (i.e. not necessarily at the minimum of the potential), except where $W = 0$, and can be easily proved using the gauge invariance of W and the general form of the F-part and the D-part of the scalar potential, given by

$$\begin{aligned} V_F &= e^K \left(K^{I\bar{J}} D_I W D_{\bar{J}} \bar{W} - 3|W|^2 \right) \quad , \\ V_D &= \frac{1}{2} \text{Re}(f_a) D^a D^a = \frac{1}{2\text{Re}(f_a)} (i\eta_a^I K_I)^2 \quad . \end{aligned} \quad (2.8)$$

($K^{I\bar{J}}$ is the inverse of the $K_{I\bar{J}} = \partial^2 K / \partial \Phi_I \partial \bar{\Phi}_{\bar{J}}$ matrix.) As mentioned above, when *just* the F-part of the KKLT potential (which coincides with the BKQ one) is minimised, the AdS SUSY-preserving minimum is at $D_I W = 0$, $W \neq 0$. If one then plugs V_D in, as done by BKQ, eq. (2.7) guarantees that that point in field space is still stationary with $D = V_D = 0$. Hence adding V_D does not lift the AdS minimum.

More generally, it is clear that, if the set of equations $D_I W = 0$ can be solved simultaneously, this corresponds to an AdS (if $W \neq 0$) SUSY minimum. Then the previous argument applies and V_D does not act as an uplifting potential.²

The results of BKQ and the argument of Choi et al. seem contradictory in appearance: if Choi et al. are right, BKQ should not have found an uplifted dS minimum once the V_D piece is added. This contradiction is solved once gauge invariance is properly implemented, as we discuss in the next section.

3. Implementation of gauge invariance and its consequences

From the general form of V_D in (2.8), it is clear that a FI term such as that of eq. (2.5) implies that the T modulus is charged under $U(1)_X$, that is, $\eta_X^T \neq 0$. More precisely, from eqs (2.5) and (2.8), we can write the auxiliary D_X field as

$$D_X = -g_X^2 \left(\frac{E}{T_R} + \sum_I q_i |\varphi_i|^2 \right) = ig_X^2 \eta_a^I K_I \quad , \quad (3.1)$$

implying

$$\eta_X^T \equiv i \frac{\delta_{GS}}{2} = \frac{-2iE}{3} \quad . \quad (3.2)$$

²However, even in that case there could still be local dS minima, which might be good enough for phenomenological purposes; this might also work just without the aid of any V_D potential or similar.

This indicates that the non-perturbative superpotential term Ae^{-aT} in (2.2) is not $U(1)_X$ gauge invariant, which is the reason why the BKQ potential does not obey the Choi et al. argument.³ The implementation of gauge invariance turns out to be crucial, not only for the consistency of the approach, but also for the qualitative features of the results. We will see also that many interesting characteristics of the potential and its minima can be predicted just on grounds of gauge invariance.

The analysis of this section and the next one, although described in terms of the type IIB set-up used by KKL^T and BKQ, is valid for any SUGRA model, in particular for those coming from heterotic string constructions.

The first step is to write the Kähler potential and the superpotential in a gauge-invariant form. For the Kähler potential, K , we follow the usual prescription

$$K = -3 \log(T + \bar{T}) \quad \longrightarrow \quad K = -3 \log(T + \bar{T} + \delta_{GS}V) \quad , \quad (3.3)$$

where $i\frac{\delta_{GS}}{2} \equiv \eta_X^T$ and V is the $U(1)_X$ vector superfield, while the non-perturbative superpotential should be written as

$$W_{np} = Ae^{-aT} \quad \longrightarrow \quad W_{np} = J(\varphi_i)e^{-aT} \quad , \quad (3.4)$$

where $J(\varphi_i)$ is an analytic function transforming under $U(1)_X$ just opposite to e^{-aT} , i.e. $J(\varphi_i) \rightarrow e^{ia\delta_{GS}/2} J(\varphi_i)$, and invariant under the other gauge groups.⁴

The first conclusion is that one of the alternative assumptions of BKQ to get $\langle \varphi_i \rangle = 0$, namely to suppose $W \neq W(\varphi_i)$, cannot occur in practice. Next, we analyse the other alternative, i.e. the possible non cancellability of the FI potential due to the signs of the $U(1)_X$ charges, which is an interesting problem on its own.

A sufficient condition for the cancellability of the D-terms in a supersymmetric theory [12–14] is the existence of an analytic function, $\tilde{J}(\varphi_i)$, invariant under all the gauge groups except the anomalous $U(1)_X$, and having an anomalous charge with sign opposite to the φ_i -independent term in D_X , i.e. E/T_R in eq. (3.1). Now, from the above transformation properties of $J(\varphi_i)$ (assuming positive a , as usual) and eq. (3.2), it is clear that $J(\varphi_i)$ precisely fulfils the previous requirements, i.e. we can identify $J(\varphi_i) \equiv \tilde{J}(\varphi_i)$ in eq. (3.4). This means that there is a set of $\langle \varphi_i \rangle$ values that cancel all the D-terms, including the anomalous one. In other words, the BKQ assumption that the anomalous FI term cannot be cancelled is not consistent with the presence of a non-perturbative *analytic* superpotential.

³If the $U(1)$ symmetry is a (gauged) R-symmetry, the non-perturbative superpotential can transform with the correct phase under a $U(1)$ transformation. Then the FI term is purely constant [10]. Notice, however, that the constant W_0 in eq. (2.2) does not transform (as it should). The possibility of using a $U(1)$ R-symmetry of this kind, with $W_0 = 0$, to get dS vacua, thanks to the interplay between V_F and V_D , has been explored in ref. [11].

⁴Incidentally, notice that an exponential for the T -superpotential is essentially the only functional form that can be made gauge invariant by the action of the matter superfields (which transform as a phase under $U(1)_X$), which is a remarkable fact. Note also that for $W_0 \neq 0$ the only option consistent with gauge invariance is to make W and K separately gauge invariant (rather than invariant up to Kähler transformations).

The apparent conclusion from the previous paragraphs is that the BKQ proposal of using FI terms to uplift the potential is definitely hopeless. However, things are more subtle in practice. W_{np} is typically an effective superpotential, so its analyticity does not need to be guaranteed in the whole range of the φ_i fields. Actually, this is the case when W_{np} is originated by gaugino condensation. For example, for $SU(N)$ with N_f “quark” pairs, $\{Q_j, \bar{Q}_j\}$, $W_{np}^{(eff)}$ is as in eq. (3.4) with [15, 16]

$$J(\varphi) = (N - N_f)(\det M^2/2)^{\frac{-1}{N-N_f}}, \tag{3.5}$$

where $(M^2)_j^i = 2Q^i\bar{Q}_j$. The important point is that the exponent of $\det M^2$ in eq. (3.5) is *negative* (at least for $N_f < N$). Therefore, $J(\varphi)$ cannot play the role of the *analytic* function $\tilde{J}(\varphi)$ in the above condition for the cancellation of the D-terms. $J(\varphi)^{-1}$ (or, more precisely, $J(\varphi)^{-(N-N_f)} \sim \det(Q\bar{Q})$) might do it, but it has the wrong sign for the cancellation of D_X . In consequence, with this $W_{np}^{(eff)}$ it is possible, in principle, that the D_X term cannot be cancelled by any choice of $\langle\varphi_i\rangle$. This rescues the BKQ assumption.

Nevertheless, even in this case, the fact that W_{np} depends crucially on φ_i alters significantly the BKQ analysis. In particular, $\langle\varphi_i\rangle$ cannot be simply approximated by zero (which presents a singularity) or replaced by a constant in W_{np} . As will be clear in sect. 6, these fields contribute their own (sizeable) part to the effective potential and cannot be ignored for minimisation issues. Consequently, the BKQ analysis must be redone incorporating the φ_i fields. We leave this task for section 6 and discuss next other relevant implications of gauge invariance.

4. Further implications for gaugino condensation and FI terms

We have just seen that a non-perturbative superpotential produced by gaugino condensation may be compatible with a non-cancellable anomalous FI term, thus rescuing the BKQ hypothesis. It is interesting, however, to point out the circumstances under which this *cannot* happen.

First of all, it is quite obvious that, if the condensing group has no matter representations, the condensing superpotential is not only incompatible with a non-cancellable FI term, but with the existence of a FI term at all.⁵ This occurs because the presence of the FI term indicates a non-trivial transformation of the T -field, which can only be compensated in the non-perturbative superpotential (3.4) by the transformation of $J(\varphi)$, which is a function of the matter representations, as given in eq. (3.5).

Second, if the fields transforming under the condensing group are massive, gaugino condensation will be again incompatible with a non-cancellable FI term. By ‘massive’ we mean that the superpotential contains operators with the form of a mass term, with (generically) field-dependent masses. For example, ordinary Yukawa operators are mass terms from this point of view. Since in the gaugino condensation literature the massive

⁵Blanco-Pillado et al. [17] have argued that, in the presence of D- anti D-brane pairs, the four dimensional effective action should also include a constant FI term coming from the tension of the branes. Here we do not consider such a possibility.

case is very common, it is appropriate to discuss this point more in depth. Working again with $SU(N)$ with N_f $\{Q, \bar{Q}\}$ pairs, if these fields are massive the superpotential contains a term

$$W \supset [\mathcal{M}(A)]^{ij} Q_i \bar{Q}_j \quad , \quad (4.1)$$

where A represents other chiral fields (with possible VEVs, but this is not relevant in the following discussion). The gauge invariance of W implies that $\det[\mathcal{M}(A)]$ transforms under $U(1)_X$ exactly opposite to $\det[Q\bar{Q}] \propto \det M^2$. Therefore we can simply use $\det[\mathcal{M}(A)]$ as the $\tilde{J}(\varphi)$ function of the cancellation condition [its charge has the correct sign under $U(1)_X$]. This guarantees the existence of $\langle A \rangle$ values cancelling the FI D-term.

This statement is confirmed by the fact that, in the massive case, the chiral superfields can be integrated out, so that $W_{np}^{(eff)}$ reads [16, 18, 19]

$$W_{np}^{(eff)} \propto [\det \mathcal{M}(A)]^{\frac{1}{N}} e^{-aT} \quad . \quad (4.2)$$

This has the form (3.4), with $J(\varphi)$ similar to eq. (3.5), but now with a positive exponent. Therefore we can use $\det \mathcal{M}(A)$ as the analytic $\tilde{J}(\varphi)$ function of the D-cancellation condition.⁶

On the other hand, many authors have argued that the limit $m \rightarrow 0$ is singular [16], or it leads to the disappearance of an effective supersymmetric superpotential [15]. According to that, it could seem that the only acceptable case is the massive case and, in consequence, we would be back to the strong statement that the D-term must be necessarily cancelled by some choice of $\langle \varphi_i \rangle$. However, as we discuss next, those arguments can be revisited to see that the formulation with massless matter is not problematic in this case.

Based on arguments related to the Witten index for global SUSY [20, 21], the authors of the previous references have argued that the $m \rightarrow 0$ case cannot be described by a non-trivial superpotential with SUSY preserving vacuum. In particular, for a large class of global SUSY scenarios, Affleck et al. [15] have shown that, for $m \rightarrow 0$, the minimisation of the potential leads to $D_M W \rightarrow 0$, and hence to $M^2 \rightarrow \infty$ (run-away behaviour) and $W_{np} \rightarrow 0$. This indicates that there is no effective SUSY superpotential describing the condensation of gauginos in that case, which is consistent with the mentioned general expectations.

In our case we have some differences with the previous scenarios: a Supergravity framework, the presence of the constant flux piece, W_0 , and the dependence on the T -field. Although we see the mentioned behaviour in the global SUSY limit, in the complete case things are different. Now we have two conditions for preserving SUSY, $D_M W = 0$ and $D_T W = 0$. If both could be satisfied at the same time, then we would have a SUSY minimum. [Notice that this would not be changed by the inclusion of the FI term since, from eq. (2.7), this would cancel at that point.] Then, we would violate the Witten index

⁶It has been argued elsewhere [19] that the general form of $W_{np}^{(eff)}$ for a generic condensing group (not necessarily $SU(N)$ with N_f flavours) is $W_{np}^{(eff)} \propto [m(A)]^{3(1-\tilde{\beta}/\beta)} e^{-3/2g^2\beta}$, where β ($\tilde{\beta}$) is the β -function of the gauge group with N_f ($N_f = 0$) flavours. Again the exponent is positive, guaranteeing the cancellation of the D-term.

arguments, at least for a minimum with zero vacuum energy (notice that, for unbroken SUSY, this requires $\langle W \rangle = \langle \partial_I W \rangle = 0$, which includes the condition for unbroken SUSY in the global limit). However, if the two conditions *cannot* be satisfied at the same time, then SUSY would be necessarily broken, recovering the agreement with the general arguments. So we can have a consistent, non-singular, massless situation, provided SUSY is broken (by $D_M W \neq 0$ and/or $D_T W \neq 0$).

It is remarkable that the same conclusion can be reached using our arguments based on the cancellability of the D-terms: if we are in a theory with a non-cancellable D-term (which, as argued, is only compatible with gaugino condensation in the massless matter case), then the equations $D_I W = 0$ (I running over all the chiral superfields) and $W \neq 0$ cannot be fulfilled simultaneously. Otherwise we would have a paradox since, at that point, V_D should also be zero because of eq. (2.7), but this is not possible by hypothesis. Therefore the only way out is that either the set of equations $D_I W = 0$ (i.e. $D_M W = 0$ and $D_T W = 0$ in the simplest set-up) has no solution, which is the conclusion reached in the previous paragraph; or $W \rightarrow 0$, indicating a run-away behaviour and, therefore, the disappearance of the non-perturbative superpotential which describes the gaugino condensation.

In section 6, we will show explicitly with examples how these results take place in practice.

5. Aspects of type IIB and the Heterotic cases

Type IIB. As mentioned above, a relevant ingredient of the (type IIB) set-up of KKLT is the presence of a $SU(N)$ condensing group arising from stacks of N D7-branes wrapped on some 4-cycle of the Calabi-Yau space. It should be noticed here that for each $SU(N)$ there typically appears a $U(1)$ factor (although this is not a strict rule). Some of these $U(1)$ s, or combinations of them, can be anomalous.

Although the discussion is easier in terms of an overall (Kähler) modulus, T , in general there will be several relevant moduli, T_i , corresponding to the independent 4-cycles. The gauge kinetic functions of the different gauge groups, f_a , are combinations of these moduli, with coefficients that depend on the geometric structure of the associated 4-cycle. In other words, the kinetic terms for the gauge superfields are of the form

$$\mathcal{L} \supset \int d^2\theta \frac{1}{8\pi} \sum_i k_a^{(i)} T_i W_a^\alpha W_{a\alpha} \quad , \quad (5.1)$$

where W_a^α is the field strength superfield of the G_a gauge group and $k_a^{(i)}$ are positive, $\mathcal{O}(1)$ model-dependent constants [22]. Note that eq. (5.1) includes axionic-like couplings $\sim k_a^{(i)} (\text{Im } T_i) F_a \tilde{F}_a$. Likewise, there may be couplings of the $\text{Im } T_i$ to $R\tilde{R}$ (R denoting the 4D Riemann tensor) which depend on the particular geometrical structure of the model. As usual, if the T_i -fields transform non-trivially, $T_i \rightarrow T_i + i \frac{\delta_{GS}^{(i)}}{2} \epsilon$, under a particular $U(1)$, say $U(1)_X$, this will be reflected in the presence of T_i -dependent FI terms. The corresponding transformation of the Lagrangian (5.1) is $\propto k_a^{(i)} \delta_{GS}^{(i)} F_a \tilde{F}_a$, which has to be compensated, if different from zero, by the transformation arising from the $[G_a]^2 \times U(1)_X$ anomaly. This

is the Green-Schwarz mechanism. More precisely, for $G_a \neq U(1)_X$ this requires

$$\sum_i k_a^{(i)} \delta_{GS}^{(i)} = -\frac{1}{\pi} \sum_r K(r) q_X(r) , \tag{5.2}$$

where r runs over all the chiral superfields transforming under G_a , and $K(r)$, $q_X(r)$ are the corresponding representation index and $U(1)_X$ charge, respectively. For $G_a = U(1)_X$ there is an extra (1/3) factor

$$\sum_i k_X^{(i)} \delta_{GS}^{(i)} = -\frac{1}{3\pi} \sum q_X^3 , \tag{5.3}$$

where the second sum runs over all the states. If G_a is a different $U(1)$, say $U(1)_a$, the (1/3) factor disappears and q_X^3 is replaced by $q_a^2 q_X$. Similarly, the mixed gauge-gravitational anomaly, proportional to $\sum q_X$ gets cancelled by the (model-dependent) couplings of the moduli to $R\tilde{R}$. All the remaining gauge anomalies (e.g. $U(1)_1 \times U(1)_2 \times U(1)_X$) are vanishing.

If there is just one overall-modulus, T , the gauge group is typically $SU(N) \times U(1)$. For several T_i , there can be many $SU(N) \times U(1)$ factors, with different values of $k_a^{(i)}$, $\delta_{GS}^{(i)}$. The Standard Model gauge group could arise in this way from wrapped D7-branes or, alternatively, from stacks of D3-branes sitting at a point of the compactified space. In the latter case the gauge coupling is given by the dilaton S , instead of T .

In summary, although conditions (5.2, 5.3) for anomaly cancellation must be fulfilled, and they are crucial for the consistency of the approach, there is a lot of freedom (i.e. model-dependence) for the possible values of the charges and the $k^{(i)}$ coefficients.

Let us give, for future use, the relevant formulae for the overall modulus case and an $SU(N) \times U(1)_X$ gauge group with N_f quark pairs, $\{Q_j, \bar{Q}_j\}$, with charges q and \bar{q} respectively. For simplicity we will consider also an overall squark condensate $|M|^2 \equiv |M_1|^2 = |M_2|^2 = \dots$, where $M_i = \sqrt{2Q^i \bar{Q}_i}$, with $i = 1, \dots, N_f$. We are assuming here $|Q^i|^2 = |\bar{Q}_i|^2 \equiv |Q|^2$, which guarantees the cancellation of the $SU(N)$ D-term.

The Kähler potential and the gauge kinetic functions for this system are given (in M_p units) by

$$K = -3\log(T + \bar{T}) + \sum_{i=1}^{N_f} (|Q_i|^2 + |\bar{Q}_i|^2) = -3\log(T + \bar{T}) + N_f |M|^2 , \tag{5.4}$$

$$f_N = \frac{k_N}{2\pi} T, \quad f_X = \frac{k_X}{2\pi} T , \tag{5.5}$$

where k_N , k_X are $\mathcal{O}(1)$ positive constants. The previous equation assumes a minimal Kähler potential for Q_i , \bar{Q}_i , which is a simplification, but not important for the present discussion, nor for the results of the next section. Under a $U(1)_X$ transformation (with parameter ϵ) T transforms as $T \rightarrow T + i \frac{\delta_{GS}}{2} \epsilon$. The anomaly cancellation conditions for

$SU(N)^2 \times U(1)_X$ and $U(1)_X^3$ read⁷

$$\delta_{GS} = -\frac{N_f(q + \bar{q})}{2\pi k_N} = -\frac{NN_f(q^3 + \bar{q}^3)}{3\pi k_X}, \quad (5.6)$$

where we have supposed for simplicity that, beside the quarks, there are no other fields with non-vanishing $U(1)_X$ charge (otherwise, the relations are straightforwardly modified). The effective condensation superpotential is given by

$$\begin{aligned} W_{np} &= (N - N_f) \left(\frac{2^{N_f} \Lambda^{3N - N_f}}{M^{2N_f}} \right)^{\frac{1}{N - N_f}} \\ &= (N - N_f) \left(\frac{2^{N_f}}{M^{2N_f}} \right)^{\frac{1}{N - N_f}} e^{\frac{-4\pi k_N T}{N - N_f}} \\ &= (N - N_f) \left(\frac{2^{N_f}}{M^{2N_f}} \right)^{\frac{1}{N - N_f}} e^{\frac{2N_f(q + \bar{q})T}{\delta_{GS}(N - N_f)}}, \end{aligned} \quad (5.7)$$

where in the last line we have used eq. (5.6). Notice that W_{np} has the form (3.4) and is indeed gauge invariant, as desired. In other words, the anomaly cancellation condition guarantees the gauge invariance of W_{np} . Incidentally, note that there is no problem in having a 'racetrack' scenario [24–27], i.e. several condensation superpotentials with different exponents, since all of them are rendered gauge-invariant thanks to the presence of matter fields. The D-part of the scalar potential is obtained from the generic expression eq. (2.8)

$$\begin{aligned} V_D &= \frac{\pi}{4k_X T_R} \left(N_f(q + \bar{q})|M|^2 - \frac{3\delta_{GS}}{2T_R} \right)^2 \\ &= \frac{\pi}{8k_N T_R} \frac{3(q + \bar{q})}{N(q^3 + \bar{q}^3)} \left(N_f(q + \bar{q})|M|^2 - \frac{3\delta_{GS}}{2T_R} \right)^2. \end{aligned} \quad (5.8)$$

Heterotic. For the heterotic string case things are much more constrained. The gauge kinetic functions are given by the dilaton field, $S = S_R + iS_I$, up to Kac-Moody level factors, k_a (positive integer-numbers, except for $U(1)$ groups) and one-loop string corrections,

$$\mathcal{L} \supset \int d^2\theta \frac{1}{4} \sum_i k_a^{(i)} S W_a^\alpha W_{a\alpha}. \quad (5.9)$$

In consequence there can be only one anomalous $U(1)$, whose anomaly is cancelled by the transformation of S , which thus triggers an FI term as explained above [28]. For example, in conventional Calabi-Yau and orbifold compactifications with $k_a = 1$ (except for $U(1)$ groups), the anomaly cancellation condition reads [28, 29]

$$-\delta_{GS} = \frac{1}{2\pi^2} \sum_r K(r) q_X(r) = \frac{1}{2\pi^2} \frac{1}{3k_X} \sum_n q_X^3 = \frac{1}{2\pi^2} \frac{1}{k_a} \sum_n q_X q_a^2 = \frac{1}{2\pi^2} \frac{1}{24} \sum_n q_X, \quad (5.10)$$

where the notation is as for eqs (5.2), (5.3), q_a are the charges of the states under any $U(1)_a \neq U(1)_X$ gauge group, and n runs over all the states.

⁷The sign of the right hand side of eq. (5.6) differs from the one quoted in [23] (in the context of heterotic string effective Supergravity) since our convention for the quark charges is that, under a $U(1)_X$ transformation, $Q \rightarrow e^{iq\epsilon} Q$, $\bar{Q} \rightarrow e^{i\bar{q}\epsilon} \bar{Q}$, while in their (implicit) conventions the sign of these exponents was negative.

The anomalous D-potential associated with $U(1)_X$ has the form

$$\begin{aligned}
 V_D &= \frac{1}{8k_X S_R} \left(\sum_n q_x^{(n)} K_i \phi_i - \frac{\delta_{GS}}{4S_R} \right)^2 \\
 &= \left(\frac{\sum_n q_X}{\sum_n q_X^3} \right) \frac{1}{S_R} \left(\sum_n q_x^{(n)} K_n \phi_n - \frac{\sum_n q_X}{192\pi^2} \frac{1}{S_R} \right)^2, \tag{5.11}
 \end{aligned}$$

where $q_x^{(n)}$ is the anomalous charge of ϕ_n and the explicit form of $K_n = \partial K / \partial \phi_n$ depends on the untwisted or twisted character, and the corresponding modular weight, of the ϕ_n -field, being in general a function of the Kähler moduli [30]. (In the minimal Kähler simplification, $K_i = \phi^+$.)

The formulation of the gaugino condensation superpotential is similar to the type IIB case. For a $SU(N) \times U(1)_X$ gauge group with N_f quark pairs, $\{Q_j, \bar{Q}_j\}$, with charges q and \bar{q} respectively, and in the overall squark condensate simplification, $|M_1|^2 = |M_2|^2 = \dots \equiv |M|^2$ [$M_i = \sqrt{2Q^i \bar{Q}_i}$, with $i = 1, \dots, N_f$], the effective condensation superpotential is given by

$$\begin{aligned}
 W_{np} &= (N - N_f) \left(\frac{2^{N_f} \Lambda^{3N - N_f}}{M^{2N_f}} \right)^{\frac{1}{N - N_f}} \\
 &= (N - N_f) \left(\frac{2^{N_f}}{M^{2N_f}} \right)^{\frac{1}{N - N_f}} e^{\frac{-8\pi^2 k_N S}{N - N_f}} \\
 &= (N - N_f) \left(\frac{2^{N_f}}{M^{2N_f}} \right)^{\frac{1}{N - N_f}} e^{\frac{2N_f(q + \bar{q})S}{\delta_{GS}(N - N_f)}}. \tag{5.12}
 \end{aligned}$$

For the heterotic string, it is not possible to play freely with a constant superpotential triggered by fluxes [8], so in order to generate realistic non-trivial minima for S one needs more than one condensing groups. This is the so-called 'racetrack mechanism' [24–27] (for other mechanisms to stabilise the dilaton see, for example, refs [31–35]). Then, suppose that the gauge group is $SU(N_1) \times SU(N_2) \times U(1)_X$. For each condensate there is an effective superpotential of the form (5.12). Assuming that, beside the quarks, there are no other matter fields with non-vanishing q_X -charges, the anomaly cancellation conditions (5.10) read now, with an obvious notation,

$$\begin{aligned}
 -\delta_{GS} &= \frac{1}{4\pi^2} N_{f1}(q_1 + \bar{q}_1) = \frac{1}{4\pi^2} N_{f2}(q_2 + \bar{q}_2) \\
 &= \frac{1}{6\pi^2 k_X} \sum_{j=1,2} N_j N_{fj} (q_j^3 + \bar{q}_j^3) = \frac{1}{48\pi^2} \sum_{j=1,2} N_j N_{fj} (q_j + \bar{q}_j). \tag{5.13}
 \end{aligned}$$

(If there are extra fields, the modification of these relations is straightforward.) Then V_D has the form (5.11) with $\sum_n q_x = \sum_{j=1,2} N_j N_{fj} (q_j + \bar{q}_j)$, $\sum_n q_x^3 = \sum_{j=1,2} N_j N_{fj} (q_j^3 + \bar{q}_j^3)$.

It is worth-noticing that, in the absence of extra matter, the anomaly cancellation condition forces $N = 12$, independently of the number of condensing groups. However, even in this case, a racetrack is possible, since the number of flavours can be different for each one.

6. Examples in type IIB

We will now illustrate explicitly the results of the previous sections, in particular the realisation of uplifting D-terms in the context of (N=1, D=4) effective supergravities from (type IIB) string theory. The scenario can be considered as a simple, gauge invariant version of the BKQ model which cures previous inconsistencies and allows for dS vacua.

6.1 The structure of the scalar potential and SUSY breaking

Let us consider the KKLТ set-up with gaugino condensation as the origin of the non-renormalisable superpotential. We will work in the simplest case of a single overall modulus, T , and $SU(N) \times U(1)_X$ gauge group with N_f quark pairs, $\{Q_j, \bar{Q}_j\}$, with anomalous charges q and \bar{q} respectively. We will also use an overall condensate $|M|^2$, as discussed in the previous section. There could be a singlet φ as well, giving mass to the condensate but, as argued in section 4, we are omitting it in order to allow for a non-cancellable D-term.⁸ The Kähler potential and the gauge kinetic functions are thus given by eqs. (5.4), (5.5), and the superpotential reads

$$W = W_0 + W_{np} , \tag{6.1}$$

where W_0 is the effective constant superpotential triggered by the presence of fluxes and W_{np} is the gauge-invariant gaugino condensation superpotential, given by eq. (5.7).⁹ The scalar potential $V = V_F + V_D$ can be explicitly written using the general formulae (2.8). In particular, V_D is given by eq. (5.8).

Let us first show that this simple model breaks SUSY, both by F- and D-terms, which opens the possibility of a dS vacuum thanks to the V_D contribution. The F-auxiliary components of the T and M superfields are proportional to the Kähler derivatives. In this case, $F_{T(M)} = \exp(K/2)D_{T(M)}W$, with

$$\begin{aligned} D_T W &= W_T + K_T W = -\frac{4\pi k_N}{N - N_f} W_{np} - \frac{3}{2T_R} W , \\ D_M W &= W_M + K_M W = -\frac{2N_f M^{-1}}{N - N_f} W_{np} + N_f \bar{M} W . \end{aligned} \tag{6.2}$$

The condition for unbroken SUSY by the previous F-terms is $D_T W = D_M W = 0$. One can immediately see that this requires

$$\text{Unbroken SUSY} \Rightarrow |M|^2 = -\frac{3}{4\pi k_N T_R} . \tag{6.3}$$

Notice that, substituting eq. (6.3) in V_D [given by eq. (5.8)], and using the anomaly cancellation relation (5.6), makes V_D cancel. This is a manifestation of the general connection (2.7) between F and D-terms. However, condition (6.3) *cannot* be fulfilled since the right

⁸In ref. [36] the issue of gauge invariance was addressed in the presence of massive matter condensates. The resulting formalism was applied to the derivation of soft breaking terms, in particular to estimating the contribution of the resulting D-term.

⁹For a detailed study of the interplay between both terms, including the origin of W_0 , see ref. [37].

hand side is negative definite. Therefore, whichever the minimum of V_F be, it will correspond to a SUSY breaking point, with non vanishing V_D . This breaking of SUSY by the F-terms is in contrast with the BKQ analysis, that did not consider the role of the matter fields to implement gauge invariance. On the other hand, if we had added an extra singlet φ to give mass to the quarks, V_D would have become immediately cancellable. To see this, notice that the charge of φ must have opposite sign to $(q + \bar{q})$ (and thus to $-\delta_{GS}$), so an appropriate VEV for φ is able to cancel V_D . This confirms the results of section 4 concerning the impossibility of having a non-cancellable V_D if the quark fields have effective masses.

Let us now investigate in more detail the structure of the scalar potential. For the remainder of this analysis it is particularly convenient to split the complex fields T and M in the following way

$$\begin{aligned} T &= T_R + iT_I, \\ M &= |M|e^{i\alpha_M}. \end{aligned} \tag{6.4}$$

Then eqs. (6.2) read

$$D_T W = -\frac{3W_0}{2T_R} - \left(\frac{3(N - N_f)}{2T_R} + 4\pi k_N \right) \left(\frac{2^{N_f}}{|M|^{2N_f}} \right)^{\frac{1}{N - N_f}} e^{\frac{-4\pi k_N T_R}{N - N_f}} e^{\frac{-2i(N_f \alpha_M + 2\pi k_N T_I)}{N - N_f}}, \tag{6.5}$$

$$D_M W = \frac{N_f}{M} \left[|M|^2 W_0 + (|M|^2(N - N_f) - 2) \left(\frac{2^{N_f}}{|M|^{2N_f}} \right)^{\frac{1}{N - N_f}} e^{\frac{-4\pi k_N T_R}{N - N_f}} e^{\frac{-2i(N_f \alpha_M + 2\pi k_N T_I)}{N - N_f}} \right].$$

It is now straightforward to write V_F plugging these expressions into eq. (2.8). Due to the presence of the same phase, $2(N_f \alpha_M + 2\pi k_N T_I)/(N - N_f)$, in $D_T W$ and $D_M W$, V_F has the form

$$V_F = A(T_R, |M|^2) + B(T_R, |M|^2) \cos \left[\frac{2}{N - N_f} (N_f \alpha_M + 2\pi k_N T_I) \right], \tag{6.6}$$

with

$$\begin{aligned} A(T_R, |M|^2) &= \frac{e^{N_f |M|^2}}{(2T_R)^3} \left[N_f W_0^2 |M|^2 + \left(\frac{2^{N_f}}{|M|^{2N_f}} \right)^{\frac{2}{N - N_f}} e^{\frac{-8\pi k_N T_R}{N - N_f}} \left(\frac{(8\pi k_N T_R)^2}{3} \right. \right. \\ &\quad \left. \left. + 16\pi k_N T_R (N - N_f) + \frac{N_f}{|M|^2} [|M|^2 (N - N_f) - 2]^2 \right) \right] \end{aligned} \tag{6.7}$$

$$\begin{aligned} B(T_R, |M|^2) &= \frac{e^{N_f |M|^2}}{(2T_R)^3} \left(\frac{2^{N_f}}{|M|^{2N_f}} \right)^{\frac{1}{N - N_f}} e^{\frac{-4\pi k_N T_R}{N - N_f}} \\ &\quad \times 2W_0 [N_f |M|^2 (N - N_f) - 2N_f + 8\pi k_N T_R]. \end{aligned} \tag{6.8}$$

V_F has extrema for

$$\left[\frac{2}{N - N_f} (N_f \alpha_M + 2\pi k_N T_I) \right] = n\pi, \tag{6.9}$$

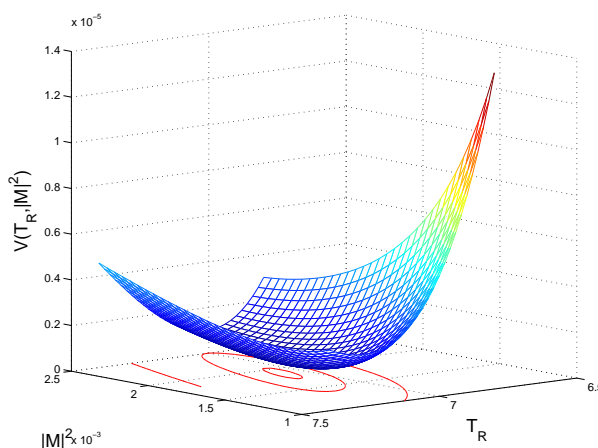


Figure 1: 3D plot of the potential as a function of T_R and $|M|^2$ for the values of the parameters shown in the text. Contour lines have been projected on the base in order to see the minimum.

with n integer. Depending on the signs of A and B , n even (odd) will correspond to a minimum (maximum) of V_F or vice versa. These are also extrema of the whole $V = V_F + V_D$ potential, since V_D , as given by eq. (5.8), does not depend on α_M, T_I . Therefore, one can integrate out one combination of these two real variables through eq. (6.9) (there is another combination which is completely flat) and work with $V(T_R, |M|^2) = V_F + V_D$, with V_F and V_D given by eq. (6.6) [with the cosine = -1 , which corresponds to the solution of minimum in our cases] and eq. (5.8) respectively.

Next we explore the minimisation of this potential, searching for examples of dS vacua.

6.2 Working examples. General characteristics

Let us start by considering the previous set up with the following choice of parameters: $N = 15$, $k_N = 1$, $N_f = 1$, $q = \bar{q} = -2$, $W_0 = 0.30$. Incidentally, $N_f = 1$ means that there is just one $M = \sqrt{2Q\bar{Q}}$ field, so we are not making any overall squark condensate assumption.

The total potential, $V_F + V_D$, as a function of the T_R and $|M|^2$ variables (the other two, α_M, T_I , have been integrated out as described above) is plotted in figure 1, where one can see the existence of a positive (dS) minimum, as desired. A contour plot has been projected on the base in order to appreciate better the position of the minimum, which happens for $T_R = 7.07$, $|M|^2 = 0.00188$ and $V_F + V_D = 5.64 \cdot 10^{-7}$. We have performed an analysis of the Hessian matrix in all four variables to confirm that this is a real minimum of V .

It is perhaps more illustrative to show the contribution of each part of the potential separately. In figure 2 we show a contour plot of V_F in the $(T_R, |M|^2)$ plane. We can clearly see that this potential has a minimum for $T_R = 5.99$, $|M|^2 = 1.94$ and $V_F = -0.00022$. We have also shown the condition $F_T = 0$ to illustrate the fact that the F-term associated to the modulus does not break SUSY at this stage. However, the remaining F-term,

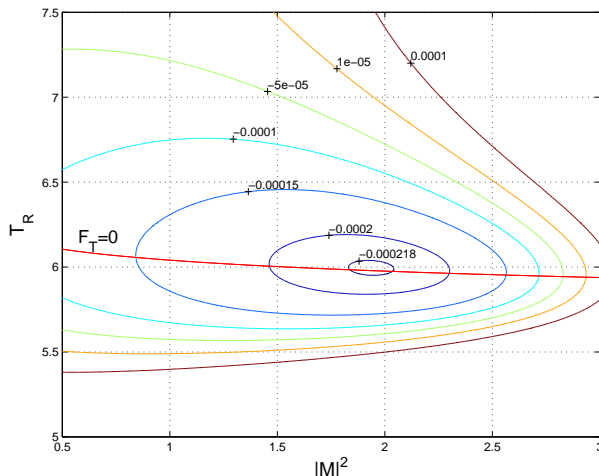


Figure 2: Contour plot of the F-potential, V_F , as a function of T_R and $|M|^2$ for the values of the parameters shown in the text. The condition $F_T = 0$ is also shown.

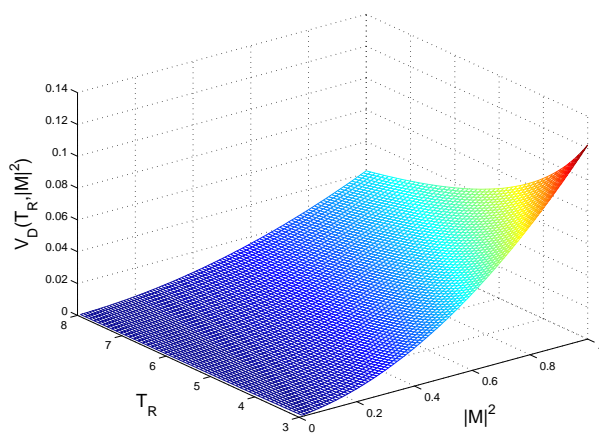


Figure 3: 3D plot of the D-potential, V_D as a function of T_R and $|M|^2$ for the values of the parameters shown in the text.

F_M , does break SUSY and, in fact, the line $F_M = 0$ lies outside the boundaries of the figure. Therefore, the F-part of the potential generates a SUSY breaking minimum with negative vacuum energy, as expected from the discussion of subsection 6.1.

As for V_D , this is shown in figure 3. Here we can see clearly that, along the $|M|^2$ direction, the minimum lies at $|M|^2 = 0$, while there is a runaway direction along increasing T_R . The sum of the two potentials is such that the minimum of $V_F + V_D$ happens for similar values of T_R as for V_F , but for much smaller values of $|M|^2$. The interplay between V_F and V_D to generate a dS minimum can be well appreciated in figure 4, where we have plotted V_F , V_D and $V = V_F + V_D$ as a function of T_R for $|M|^2$ fixed at its value in the minimum of V .

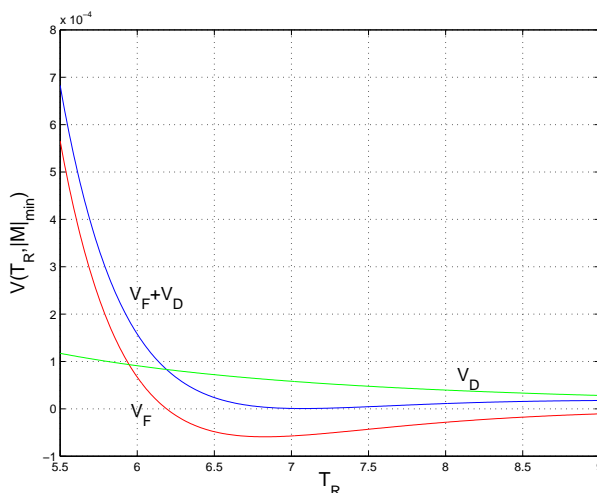


Figure 4: Plot of the F-potential, V_F (red), the D-potential, V_D (green) and the sum of both, $V_F + V_D$ (blue), as a function of T_R for the example shown in the text and $|M|^2$ fixed to its value at the overall minimum.

N	$-(q, \bar{q})$	W_0	$T_R _{\min} (T_R _{F\min})$	$ M ^2_{\min} (M ^2_{F\min})$	$V_F + V_D _{\min} (V_F _{\min})$
15	(2,2)	0.30	7.07 (5.99)	0.00188 (1.94)	$5.64 \cdot 10^{-7}$ (-0.000219)
14	(1,2)	0.25	6.83 (5.70)	0.00169 (1.94)	$7.38 \cdot 10^{-6}$ (-0.000178)
13	(2,3)	0.27	6.29 (5.09)	0.00161 (1.93)	$2.04 \cdot 10^{-5}$ (-0.000286)
12	(3,3)	0.28	5.92 (4.54)	0.00129 (1.92)	$4.52 \cdot 10^{-5}$ (-0.000423)
18	(3,4)	0.26	8.97 (7.80)	0.00134 (1.95)	$2.70 \cdot 10^{-7}$ ($-7.82 \cdot 10^{-5}$)
13	(3,-2)	0.03	8.57 (7.48)	0.00109 (1.95)	$1.34 \cdot 10^{-7}$ ($-1.32 \cdot 10^{-6}$)

Table 1: Viable examples with $N_f = 1$, $\cos = -1$ and $k_N = 1$.

We have found many more examples, with reasonable values of the parameters, where this pattern is repeated. In table 1 we present a handful of successful cases, including the one shown in the figures. The 'min' ('Fmin') subscript means that the quantity is evaluated at the minimum of V (V_F).

A number of comments are in order:

- The values found for T_R are in the 5 – 10 range (in M_p units), which is satisfactory for the Supergravity approximation we are using to be valid (which requires $T_R \gtrsim 1$), but also from the phenomenological point of view. Notice here that the value of T_R corresponds to $g_{\text{YM}}^{-2}(M_{\text{string}})$ up to $\mathcal{O}(1)$ model-dependent factors, called k_a in sect. 5. If the Standard Model (SM) gauge group arises from wrapped D7-branes (as we have assumed for the condensing gauge groups) and all $k_a = 1$, then one should get $T_R \sim 12$. This is in fact not far from the values found in the examples of table 1, where $k_N = 1$ has been taken for the condensing group, but no assumption about the k_a factors of the SM gauge groups has been made. Therefore, playing with these

$O(1)$ factors, it is clear that the quoted values of T_R are perfectly realistic. On the other hand, as discussed in sect. 5, the SM gauge group could alternatively arise from stacks of D3-branes sitting at a point of the compactified space, in which case the gauge coupling is given by the dilaton S , instead of T . Then, the value of T_R does not have a direct phenomenological meaning and, again, the presented examples are perfectly viable.

- The value of W_0 sets the overall scale of the F-potential, and, consequently, determines the scale of the non-perturbative potential, which has to be comparable in size in order to create a minimum of V_F . Given that W_0 is an effective parameter, coming from the flux stabilisation of the other moduli in the model, we treat it as a phenomenological quantity and simply state the range of values for which we find suitable minima, which, as can be seen from the table, is around $W_0 \sim 0.3$ (for both anomalous charges having the same sign). Smaller values of W_0 result in a smaller magnitude for V_F which would imply that V_D dominates and we lose the minimum. Larger values would result in a too large V_F and the uplifting by V_D would not be efficient enough.
- Playing with the value of W_0 one can find $V = 0$ (i.e. Minkowski) vacua, or adjust the cosmological constant to an arbitrary small value. E.g. for the example shown in the figures we can tune $W_0 = 0.301566$ to have a minimum at $T_R = 7.07$, $|M|^2 = 0.0019$ and $V = 1.10^{-9}$. The value $W_0 = 0.301567$ would already render a negative minimum. Roughly speaking each decimal place tuned in W_0 reduces the value of V at the minimum by half an order of magnitude.
- The small values of $|M|^2$ allow, for the purpose of minimisation, to neglect $|M|^2$ in V_D . In this way one exactly recovers the BKQ potential (2.6). However, this does not mean that one can simply replace $|M|^2$ by a constant in W_{np} and proceed as BKQ. The reason is that the contribution of the M -field to the V_F potential, $e^K |D_M W|^2 \sim k/T_R$ (where k is a constant), is sizeable and cannot be ignored in the minimisation. Likewise, the dependence of V in both fields, T and M , may be relevant to inflation.
- To obtain $T_R > 1$ at the minimum, a value of $N \geq 10$ has to be assumed. This is due to the fact that the minimum for the F-potential comes from the interplay between W_0 and W_{np} , which determines the order of magnitude of the exponent of W_{np} , $4\pi k_N/(N-1)$, in terms of W_0 and T_R . Then, for the quoted size of W_0 and $T_R > 5$, N must be at least of order 12, as shown.
- The values of the anomalous charges (q, \bar{q}) affect the scale of the D-potential and must, therefore, be balanced with the corresponding value of W_0 for the uplifting mechanism to be efficient.

- We have found $F_T \sim 0$ at the minimum of V_F in all examples. This leads to a simple expression for $|M|_{F_{\min}}^2$, given by

$$\left(\frac{2}{|M|_{F_{\min}}^2}\right)^{1/(N-1)} = \frac{\frac{3W_0}{2T_R}}{\frac{3(N-1)}{2T_R} + 4\pi k_N} e^{4\pi k_N T_R/(N-1)}. \quad (6.10)$$

For the values of parameters considered one can check that the previous equation leads to a value of $|M|_{F_{\min}}^2 \sim 2$ which is remarkably stable. As commented before, this value gets very suppressed once V_D is added.

- After adding the V_D piece, both $D_T W$ and $D_M W \neq 0$. Hence, SUSY becomes broken by the T and M F-terms, and also by the D-term associated to $U(1)_X$. The goldstino field can be written as $\eta = c_M \tilde{M} + c_T \tilde{T} + c_\lambda \lambda_X$, where \tilde{T} , \tilde{M} are the (canonically normalised) fermionic components of T , M , and λ_X is the (canonically normalised) gaugino associated to $U(1)_X$. For the example shown in the figures, the coefficients $\{|c_M|, |c_T|, |c_\lambda|\}$ are in the ratio $\{1 : 25 : 42\}$. The resulting gravitino mass is naturally too large for ordinary low-energy SUSY phenomenology. E.g. for the same example as above, $m_{3/2} = \exp(K/2)W \simeq M_p/177$. This large $m_{3/2}$ is due to the magnitude of $V_D \sim [\mathcal{O}(M_p^2)/(4\pi^2)]^2$, which has to be comparable to the absolute size of V_F in order to uplift the AdS minimum, without destroying it. This may be a phenomenological shortcoming if one attempts to get conventional low-energy SUSY. We leave some further comments on this issue for the conclusions.

In summary, playing with a constant superpotential (triggered by fluxes), a non-renormalizable superpotential originated by gaugino condensation, with quark representations, and an anomalous $U(1)$ group with non-cancellable FI D-term (which, for gauge invariance consistency, requires the quarks to be massless), it is easy to find examples where an AdS minimum is produced by the F-potential, and subsequently uplifted to dS by the D-term. There is no fine tuning in our results (unless one wishes to fine-tune the cosmological constant) and, in practice, this fixes the acceptable ranges for W_0 and the rank of the gauge group.

7. Conclusions

In this article we have revisited the proposal of BKQ [3] of using a supersymmetric D-term potential (namely a Fayet-Iliopoulos one) for the uplifting of the AdS minima found by KKLT [2] in the context of type IIB theory with fluxes. The BKQ suggestion has been criticised [4, 5] since, on general SUGRA grounds, a model with vanishing F-terms must have also vanishing D-terms, which prevents the D-part of the potential (V_D) from uplifting the SUSY-preserving minima of the F-part (V_F).

First, we have reconciled the BKQ scenario with the general SUGRA arguments, by making the former gauge invariant. This requires the inclusion of matter fields, which play a crucial role for the consistency of the approach. In this context we show that a non-perturbative superpotential $\sim e^{-aT}$ (as required by the KKLT set up) produced by

gaugino condensation is only consistent with a non-cancellable V_D (as required by the BKQ proposal) if the relevant quark representations are massless. Then the minima of V_F are necessarily SUSY-breaking, either by the moduli F-terms or by the matter ones, and the uplifting by V_D can in principle work. We discussed also the details of such effective SUGRA scenarios when they arise from type IIB or from the heterotic strings, paying special attention to the anomaly cancellation constraints.

Finally, we illustrated and applied the previous results, by finding many examples of effective SUGRA models (which can arise from type IIB strings in the KKLТ context), whose potential has positive minima for reasonable values of the moduli and matter fields. This shows how the uplifting by D-terms works in practice. Before adding the V_D piece to the potential, these examples have SUSY broken by the F-term associated to the matter fields, whereas $F_T \sim 0$. The minima are initially AdS and become dS after adding the D-terms. The corresponding values of the parameters that define the model lie within natural ranges and are not fine tuned (unless one wishes a vanishing or extremely tiny cosmological constant, which is perfectly achievable).

At the minimum of the complete potential the breaking of SUSY is triggered both by the F-terms (i.e. $F_M, F_T \neq 0$) and by the anomalous D-term ($D_X \neq 0$). The gravitino mass is $\mathcal{O}(10^{-2}M_p)$, since the natural scale of V_D (and hence of V_F at the uplifted minimum) is $[\mathcal{O}(M_p/(4\pi^2))]^2$. This large gravitino mass is an obstacle for conventional low-energy SUSY (although it is OK for inflation), since it leads to $\mathcal{O}(m_{3/2})$ soft masses in the observable sector, after gravity mediation of SUSY breaking. A possibility here is simply to give up low energy SUSY. However, a particularly appealing possibility arises from noting that, although the scalar soft masses are naturally very large in this framework, the (observable) gaugino masses are vanishing at this stage if the SM gauge group arises from stacks of D3-branes sitting at a point of the compactified space (since T does not enter the relevant gauge kinetic functions). It is amusing that this scenario (which, of course, would get radiative corrections) corresponds to the assumptions of the so-called split-SUSY models [38, 39], whose possible existence and viability has been invoked precisely in the context of landscape frameworks, as those suggested by the KKLТ set up re-visited here.

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